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Rzaev, S. F. (IR-TBRZ)

Approximation theory on compact groups. (English summary)

Wavelets and splines, 74–83, *St. Petersburg Univ. Press, St. Petersburg*, 2005.

Given a function f on a compact topological group G, the author introduces a notion of modulus of smoothness $\omega_r(f;\tau)_p$, analogous to that on Euclidean space. He further defines a linear approximation error $E_n(f)_2$ as the minimal distance (measured in the L_2-norm) between f and arbitrary linear combinations of matrix coefficients of the first n irreducible representations. Here a fixed numbering of the unitary dual $\widehat{G}$ of G is used, which is arbitrary except for the requirement that the representation degree d_n of the nth representation grow monotonically with n.

The main result of the paper is the Jackson-type inequality

$$E_n(f)_2 \le c_{n,r}\omega_r(f,1/n)_2,$$

for nonconstant $f \in L_2(G)$, where

$$c_{n,r} = \begin{cases} \sqrt{\frac{d_n}{d_n-2r}} & \text{for } d_n \ge 2\frac{1}{\sqrt{1+r}}, \\ \sqrt{\frac{1}{1+r}} & \text{for } d_n = 1. \end{cases}$$

The constant $\sqrt{\frac{d_n}{d_n-2r}}$ has already been obtained for all d_n in [H. Vaezi and S. F. Rzaev, Int. J. Math. Math. Sci. **2003**, no. 20, 1251–1260; MR1979399 (2004f:42009)], hence the main new contribution is a sharpening for the case $d_n = 1$.

{Reviewer's remarks: (1) The existence of a numbering of $\widehat{G}$ in such a way that d_n is monotonically increasing is a nontrivial restriction on G. For instance, if G is nonabelian with an infinite abelian Hausdorff quotient group, no such numbering can exist.

(2) In the proof of the main result, the author concentrates on the case $d_n = 1$, having treated the other case elsewhere. He then proceeds by assuming that G is abelian. This seems to require additional arguments, since only $d_n = 1$ *for all* n implies that G is abelian.}

{For the entire collection see MR2187108 (2006g:42001)}

Reviewed by *Hartmut Führ*